MPI-PhT/96-72
August 1996

Perturbative description of inclusive energy spectra*

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Abstract

The recent LEP-1.5 data on charged particle inclusive energy spectra are analyzed within the analytical QCD approach based on Modified Leading Log Approximation plus Local Parton Hadron Duality. The shape, the position of the maximum and the cumulants moments of the inclusive energy spectrum are well described within this model. The sensitivity of the results to the running of the coupling is pointed out. A scaling law for the one-particle invariant density $E\frac{dn}{d^3p}$ at small momenta is observed, consistently with the predictions of colour coherence in soft gluon bremsstrahlung.

*to be published in the Proceedings of the High Energy Conference on Quantum Chromodynamics (QCD '96), Montpellier, France, July 4^{th}-12^{th}, 1996, Ed. S. Narison, Nucl Phys. B (Proc. Suppl.)

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1. QCD DESCRIPTION OF INCLUSIVE PARTICLE ENERGY DISTRIBUTION

Analytical predictions of inclusive observables in jet physics can be performed at parton level in the framework of the Modified Leading Log Approximation (MLLA) of QCD[1]. In this approach, parton production in jets is described in terms of a shower evolution which intrinsically includes coherence, takes care of both leading collinear and infrared singularities and of energy-momentum conservation (recoil effect). Predictions depend on two free parameters only, i.e., the infrared cut-off at which the parton evolution is stopped, Q_0, and the effective QCD scale Λ which appears in the one-loop expression for the running coupling. To connect predictions at parton level with experimental data, Local Parton Hadron Duality (LPHD)[2,3] is taken as hadronization prescription, i.e., the inclusive hadron spectra are required to be proportional to the corresponding inclusive parton spectra, obtained by allowing the parton cascade to evolve down to a cut-off Q_0 of the order of few hundreds MeV. The whole hadronization is then parametrized in terms of only one parameter, which gives the overall normalization of the spectrum.

2. KINEMATICS OF THE SOFT REGION

The comparison of parton and hadron spectra in the soft region requires some attention. On the theoretical side, partons are taken as massless and one does not distinguish between inclusive momentum or energy spectra[1]. However, a k_t cut-off, Q_0, is introduced for regularization of the infrared and collinear singularities, and the theoretical spectrum has a kinematical cut-off at $\xi = Y \equiv \log \sqrt{s}/2Q_0$. Experimentally, the momentum and energy spectra show a different behavior at small momenta due to the effect of hadron mass; even though this difference does not affect essentially the gross features of the spectra, like for instance the peak position, it is relevant in the soft region.

To safely connect the two distributions at parton and hadron level, a prescription has been proposed in [4]: the cut-off Q_0 has been associated with an effective hadron mass[2] such that $E_h = \sqrt{p_h^2 + Q_0^2}$ and one has required:

$$D(\xi, Y) \equiv E\frac{dn(\xi_E)}{dp} \quad , \quad \xi \equiv \xi_E = \log \frac{\sqrt{s}}{2E} \qquad (1)$$

In this way, a common kinematical behaviour at hadron and parton levels is indeed obtained.

3. THE ANALYSIS OF THE SHAPE

Figure (1) shows experimental data on charged particle inclusive momentum distribution, $dn/d\xi_p$ as a function of $\xi_p = \log(\sqrt{s}/2p)$, extracted at LEP-1.5 *cms* energy by ALEPH, DELPHI and OPAL Collaborations[5]. Also shown are the theoretical predictions of the Limiting Spectrum with the Q_0 parameter taken equal to 270 MeV, as suggested in [4], and the free overall normalization factor fixed to the value $K^h = 1.31$.

The Limiting Spectrum predictions with this choice of parameters reproduce well the shape around the peak, even though small deviations, compatible with statistical uncertainties, are visible in the region around the maximum. The agreement is good in the large x_p (small ξ_p) region as well, whereas in the small x_p (large ξ_p) region, data show a tail which is not well reproduced by theoretical predictions. The tail can be described if one properly takes into account the kinematical effects discussed in the previous section. By using the relation (1) between parton and hadron spectra, the theoretical predictions for the inclusive momentum spectrum is related to the Limiting Spectrum prediction $D_q^{\lim}(\xi, Y)$ by the following relation:

$$\frac{1}{\sigma}\frac{d\sigma^h}{d\xi_p} = 2K^h \frac{p}{E} D_q^{\lim}(\xi_E, Y) \qquad (2)$$

where the factor 2 takes into account that we are considering the full e^+e^- events, whereas $D_q^{\lim}(\xi, Y)$ refers to a single quark jet.

The dashed line in Figure (1) shows the prediction of eq. (2) with $K^h = 1.34$. For $E \gg Q_0$ the difference between energy and momentum becomes negligible and $dn/d\xi_p \simeq dn/d\xi_E$, $\xi_E \simeq \xi_p$. The good agreement at small and intermediate ξ_p is then not modified. However, this correction becomes important at large ξ_p (small momentum), where the rescaled prediction closely follows the tail of the experimental data.

4. THE POSITION OF THE MAXIMUM

An easily accessable characteristic of the ξ-distribution is its maximum ξ^*. The high energy behaviour of this quantity is predicted in MLLA for the Limiting Spectrum as [1]:

$$\xi^*_{asy} \;=\; Y \left[\frac{1}{2} + \sqrt{\frac{C}{Y}} - \frac{C}{Y}\right] \qquad (3)$$

with the constant term given by $C = \frac{a^2}{16\,N_C b} = 0.2915(0.3513)$ for $n_f = 3(5)$. It is actually possible to extract numerically the position of the maximum ξ^* from the shape of the Limiting Spectrum itself; even though the asymptotic formula describes very well the energy dependence of the actual maximum, it underestimates the actual value by an (approximately) constant value of the

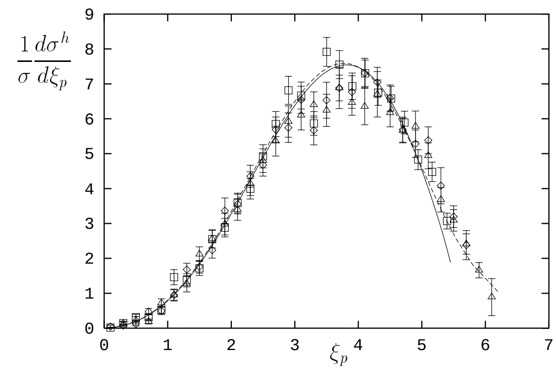

Figure 1. Charged particle inclusive momentum distribution at LEP-1.5 from ALEPH (diamonds), DELPHI (squares) and OPAL Collaborations (triangles) in comparison with theoretical predictions of the Limiting Spectrum with $Q_0 = 270$ MeV (solid line). Dashed lines show the predictions of the Limiting Spectrum after correction for kinematical effects.

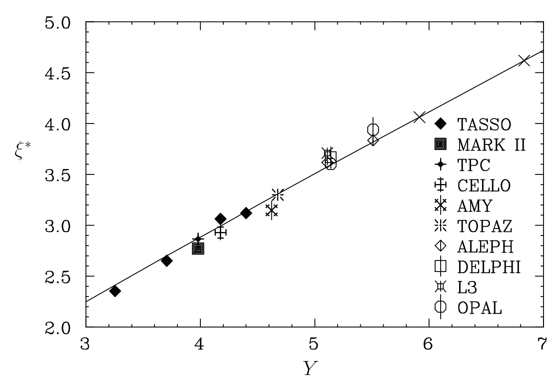

Figure 2. Maximum of the inclusive momentum distribution ξ^* as a function of Y; comparison between experimental data and theoretical prediction numerically extracted from the shape of the Limiting Spectrum (solid line); $Q_0 = 270$ MeV. Crosses are put at *cms* energies $\sqrt{s} = 200$ GeV and 500 GeV.

order of 0.1 units. This gives rise to a difference of the order of 20-30 MeV in the determination of the best value of the cut-off Q_0 from the energy dependence of the maximum.

In Figure (2) we compare experimental data on the maximum ξ^* extracted from Gaussian or Distorted Gaussian fits to the central region of the inclusive momentum spectrum with the theoretical predictions obtained extracting the maximum from the shape of the Limiting Spectrum[5]. A good agreement is found for $Q_0 = 270$ MeV and $n_f = 3$. Notice the choice of 3 active flavours, as suggested by the study of moments in [4], contrary to the usual approach, in which 5 active flavours are used. In this respect, let us remind that the number of flavours enters in the expressions for the cumulant moments through the running coupling $\alpha_s(y, n_f)$; since the MLLA is defined at one-loop level, there remains a scale ambiguity in the expression for α_s and it is then not so straightforward to fix the scale and heavy quark thresholds. As discussed in more detail in [5], kinematical reasons suggest to push the thresholds to larger scales and keep 3 active flavours only. In any case, for the maximum ξ^* the effect of different choices on the number of active flavours and on the treatment of heavy quark thresholds is of the order of 1% only.

5. MOMENT ANALYSIS

In order to study the inclusive spectrum in more detail, it is convenient to look at the energy dependence of its moments or cumulant moments[4,6]. Note that cumulants of order $q \geq 1$ are independent of the overall normalization, which is the main source of systematic uncertainties; in addition, theoretical predictions for these observables are absolutely normalized at threshold and then allow to test the perturbative picture down to low *cms* energies.

The moments $< \xi^q >$ and the cumulants K_q can be extracted from the spectrum Edn/dp as a function of ξ_E:

$$< \xi^q > \equiv \frac{1}{\bar{N}_E} \int d\xi \xi^q D(\xi) \ . \tag{4}$$

The average multiplicity $\bar{N}_E$ is obtained as the integral over ξ_E of the full spectrum Edn/dp (zeroth order moment). For the unmeasured interval

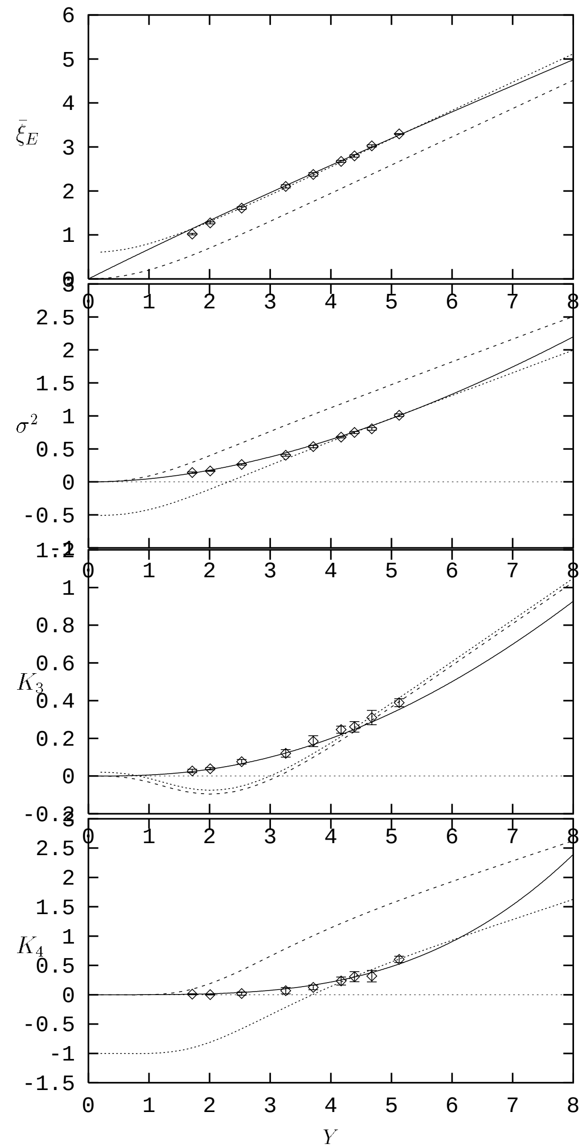

Figure 3. The first four cumulants of charged particles' energy spectra Edn/dp vs. ξ_E, are shown as a function of Y for $Q_0 = 270$ MeV. Predictions of the Limiting Spectrum of MLLA with running α_s (solid lines), of MLLA with fixed α_s ($= 0.214$) (dashed lines) and of MLLA with fixed α_s normalized by hand to the experimental point at $\sqrt{s} = 44$ GeV (dotted lines) are also shown (in all cases $n_f = 3$).

near $\xi_E \simeq Y$ (small momenta) a contribution is found by linear extrapolation as in [4].

The cumulants up to order q=4 are shown in Figure (3) as a function of Y. Theoretical predictions for the Limiting Spectrum with $Q_0 = 270$ MeV[6] are also shown (solid line). The agreement with experimental results is very satisfactory. The dashed lines show the predictions of the MLLA, but with fixed coupling α_s; this model cannot reproduce the behavior of high order cumulants, showing the sensitivity of the moments of the inclusive spectra to the running of the coupling.

It is possible to relax the absolute normalization, thus building an effective model with one more parameter for each cumulant. By rescaling the fixed α_s model to reproduce the high energy region (in particular we fixed the curve at the data point at $\sqrt{s} = 44$ GeV), one can describe the experimental data only in a small energy region (about one unit in Y) (dotted lines). Also in this case, the predictions differ from the full MLLA model with running α_s at larger *cms* energies; in this respect, the study of cumulant moments of the inclusive energy spectra at $p\bar{p}$ collisions, where larger values of Y can be reached[7], could give new and very interesting information.

6. THE BEHAVIOR OF THE INVARIANT CROSS SECTION IN THE SOFT LIMIT

Figure (4) shows the charged particle invariant cross section, Edn/d^3p, as a function of the particle energy E at different *cms* energies ranging from 3 GeV up to LEP-1.5[4,8]. The value of 270 MeV has been used for the effective mass Q_0 which enters in the kinematical relation, $E^2 = p^2+Q_0^2$. It is remarkable that all curves scale with *cms* energies within 20% at particle energy of the order of few hundreds MeV; at larger particle energies, a violation of the scaling-law is visible. In addition, all curves approach a finite limit. LEP data seems to tend to a larger limiting value; it is not clear whether this is due to a different physics at LEP energies or rather an overall systematic effect in the normalization of the different experiments.

These results are particularly interesting, since colour coherence predicts indeed that soft particles do not multiply[2]; the invariant cross-section should then be independent of *cms* energy at low particle energy and should approach a finite limit[8]. The experimental observation of this scaling law suggests that the invariant cross-section keeps trace of the effects of colour coherence present at parton level.

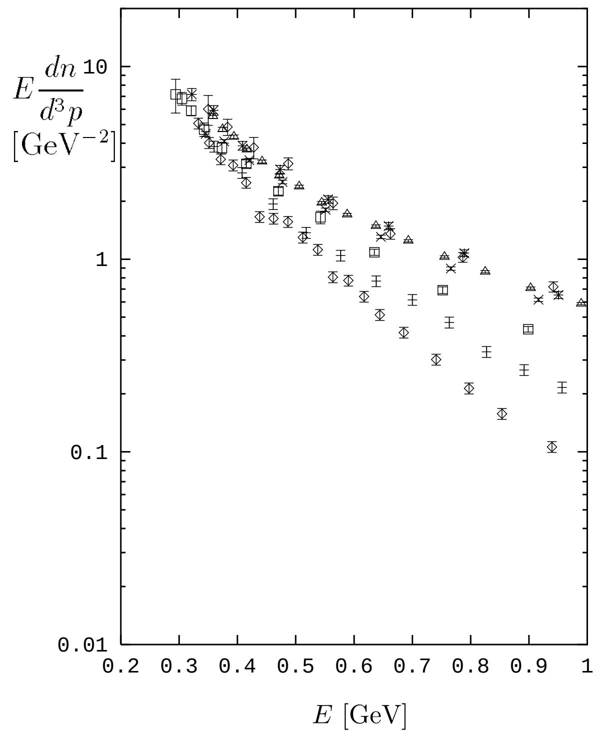

Figure 4. Invariant distribution Edn/d^3p for charged particle as a function of the particle energy E with $Q_0 = 270$ MeV.

We have considered so far the inclusive spectra for all charged particles only. It is interesting to study whether the perturbative approach can be extended to identified particle spectra, in particular π, K and p spectra. In this case, a natural mass scale would be provided by the mass of the particle itself. The success of this approach with the identification of the particle mass as the theoretical cut-off Q_0 for describing the hump-backed plateau is still controversial (see [3] for a discussion of this point). Here we concentrate on the behaviour of the invariant cross-section in the

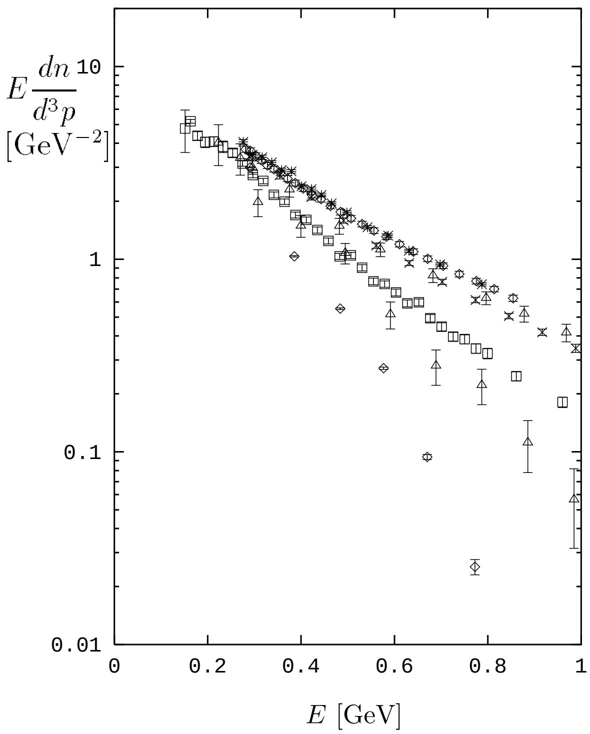

Figure 5. Same as in (4) but for charged pions with $Q_0 = m_\pi$.

soft region. Figure (5) shows the invariant cross section Edn/d^3p for charged pions as a function of the particle energy E extracted from inclusive momentum spectra at *cms* energies ranging from 1.6 GeV up to LEP-1 energy. Also in this case an approximate scaling law with *cms* energy is seen at particle energy of few hundreds MeV and a finite limit is approached. Work is in progress[8] to further investigate the consequences of this result and to extend this analysis to other particles' species.

7. CONCLUSIONS

The analysis of the recent LEP-1.5 data shows that the analytical perturbative description of inclusive particle distributions based on MLLA plus LPHD is in a good shape. The charged particle spectrum (up to the overall normalization), the position of its maximum and the cumulants moments can be described by the Limiting Spectrum with only one adjustable scale parameter $Q_0 = \Lambda$. No additional sizeable effects from hadronization are visible for these observables. The sensitivity of the inclusive particle spectra to the running of the coupling has been pointed out. The approach to an energy independent limit of the invariant density Edn/d^3p for momenta $p \to 0$ in a wide range of primary energies E_{jet} in agreement with analytical results from the QCD jet evolution which includes the coherence of the soft gluon bremsstrahlung has also been observed.

ACKNOWLEDGEMENTS

I thank Valery A. Khoze and Wolfgang Ochs for discussions and collaboration on the subjects of this talk. I thank S. Narison for the nice atmosphere created at the Conference.

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